

## Grandi's Series Applied in the Duals Method by R.S. LaFleur

The robust 'Duals method,' built-into **CertainError Uncertainty Calculator**, is based on hybrids of number and geometry and the simultaneous representation of quantities as a center point and an error vector. In the formulation, a particular situation utilizes Grandi's Series and its rendered value, G. This occurs when there is a starting point on a grid followed by a sequence of uniaxial vectors that cover an increment, in an infinite cycle (this is a scenario for quantum computing). The resulting format is

$$D = x[p] \oplus G\Delta x[v] \quad \text{where} \quad G \equiv \sum_{i=0}^{\infty} (-1)^i = 1 - 1 + 1 - 1 + \dots$$

This is applied to cover the same increment, first starting at a positive point on the grid. According to the choice of the sign on the point's scalar, the 'positive version' of the dual is

$$P = (+x)[p] \oplus (E\Delta x)[v]$$

The inverse element of this dual (for addition of dual group) is the 'negative version'

$$N = (-x)[p] \oplus (F\Delta x)[v]$$

To be an inverse, the geometric addition is null (the addition of dual group's identity, multiple zeros)

$$P \oplus N = * = 0[p] \oplus 0[v]$$

The G value is determined from this and the properties of Grandi's Series. Adding P and N, the zero-scalar on the error vector requires two versions of a common scalar, G

$$E = (-1)G$$

$$F = (+1)G$$

From this equation, F is 'done' and the E can be restated as a bias of G, by absorbing the negative one multiplier and shifting the index of the series. This relies on an invariant infinite upper limit.

$$E = (-1)G = \sum_{i=0}^{\infty} (-1)^{i+1} = \sum_{j=1}^{\infty} (-1)^j = \sum_{j=0}^{\infty} (-1)^j - 1 = G - 1$$

This is an essential step for solving G and then E (and F). By inspection,

$$G = \frac{1}{2} \quad \text{and} \quad E = -\frac{1}{2} \quad \text{and} \quad F = +\frac{1}{2}$$

Therefore the infinite cycle is equivalent to an error vector scaled to  $\pm$  half the increment

$$P = (+x)[p] \oplus \left(-\frac{1}{2}\Delta x\right)[v]$$

$$N = (-x)[p] \oplus \left(+\frac{1}{2}\Delta x\right)[v]$$